

Some Math Goodies - SMG & how CAS Can Amazingly Supercharge an algebra class

The exploration and remedial value of the eComputer Algebra System (CAS) on the TI-89 is tremendous. What graphing calculators did to the Algebra class in the past is similar to what the Symbolic Math Guide (SMG) on the TI-89 can do today. Explore and examine this astonishing pedagogical tool.

From "A Case for CAS"

[research] supported the assertion that appropriate use of CAS can enhance the teaching and learning of mathematics and lead to improved *procedural, conceptual and application performance* (Connors and Snook, 2001).

The fact that using CAS does 'no harm' to students' 'basic symbolic manipulation skills', however, does not in itself provide a complete case for using CAS. Many studies have looked at the effect of using CAS on students' mathematical skills and their understanding of concepts. *A range of studies showed that the overall conceptual understandings of the students who used a CAS were the same as or better than the understandings of their counterparts who did not use a CAS. Other studies showed that students who used a CAS better understood particular concepts – variable and function – than those who did not use a CAS. From these studies, we conclude that CAS use enables students to develop deeper conceptual understanding* " (Heid, Blume, Hollebrands and Piez, 2002).

Giving the second last word in this section to the ACDCA project, Heugl summarises nicely the advantages of using CAS. *Mathematical competence opens doors to productive futures. A lack of mathematical competence closes these doors. But the vision of mathematics teaching and learning is not the reality in the majority of classrooms and schools. In this age of information technology students need to learn a new set of mathematics basics that enables them to solve problems creatively and resourcefully. The National Council of Teachers of Mathematics in USA (NCTM) formulated guidelines for educational decisions laid out in principles and standards. One of these principles is called the "Technology Principle": Calculators and computers are reshaping the mathematical landscape, and school mathematics should reflect these changes. Students can learn more mathematics more deeply with appropriate and responsible use of technology. The investigations of the Austrian CAS projects justify an optimistic point of view. The major results are not especially mathematical contents - it is a more pupil centred, experimental way of learning. In other words, the new tool supports all of the four key qualifications.... CAS-supported mathematical education supports and encourages the 4 key qualifications:*

- *Subject competence*
- *Methodological competence*
- *Social competence*
- *Personal competence*

much better than traditional mathematics education (Heugl, 2000).

An Introduction to the *Symbolic Math Guide*

Purpose of Symbolic Math Guide (SMG): To help students learn algebra and some aspects of calculus by guiding them as they develop correct textbook-like solutions. SMG can be used when a student first learns a topic or as quick review.

SMG is a tool for learning algebra that provides additional help to support textbooks and teachers. It is not a powerful computer algebra system that can solve large classes of difficult problems. If you wish to expand $(x + y)^{15}$ or do other complicated problems, you should use the Home screen of the TI-89, TI-92 Plus, Voyage™ 200, TI-Interactive!™, or Derive™.

Features:

- Work is organized by problem sets. *SMG* comes with sample problems. Teachers, publishers, and students can create their own sets.
- Students work problems by choosing from problem-solving options that *SMG* provides based on the problem and the current solution step.
- Students learn strategy in the absence of arithmetic mistakes and lower level algebra.
- Students learn to think about math objects, how to classify them, and what transformations apply to them at various stages of solving problems.
- The transformations suggested are designed to focus on the new material being learned.

A Brief Guided Tour

You will need:

- TI-89/TI-92 Plus or Voyage™ 200 with the Advanced Mathematics operating system version 2.08 or higher)
- Symbolic Math Guide (ver 1.5 or higher).
- Problem Set Tourps.

Let's start solving problems with Symbolic Math Guide!

1. Press **ON** to start the calculator.
2. Press **APPS** and highlight **Symbolic Math Guide**.
3. Press **ENTER** **ENTER**.
4. Choose **Open** and then press **ENTER**.
5. Open **Tourps**, which contains the problems we will use for this guided tour. Note that the first problem is ready for you to work.

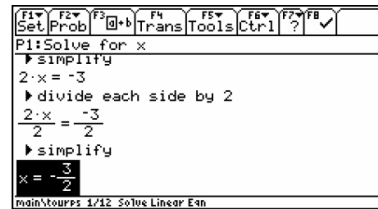
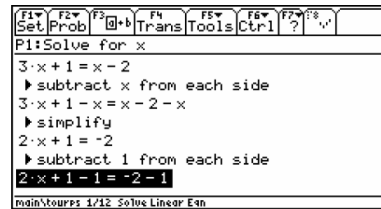
MAIN PROCESS: For each step

- A. Think about what you want to do next.
- B. Press **[F4]** to see a list of possible transformations.
- C. If you see what you want to do, press the corresponding number and follow the instructions. Anticipate the result produced by your choice.
- D. If you don't see what you want to do, choose the best choice offered or choose a different part of the current math object (See below after Problem 5.).
- E. If you need help understanding the goal of solving a problem, press **[F7]:1**.

Problem 1: Solve linear equation $3 \cdot x + 1 = x - 2$ for x .

SMG feature: For maximum learning, students should anticipate the result of each transformation. Writing each step down with pencil and paper helps some students. When in the TIME TO THINK mode, *SMG* gives students extra time to write their next step by waiting until they press **[ENTER]** to show the result. Students will probably not want to use this mode when they are using *SMG* to review material previously learned. To turn TIME TO THINK mode ON and OFF, go to Format screen (**[F1]:9**).

1. Press **[F4]** and choose a transformation that will accomplish this goal. For example, press **[F4]** and choose **subtract ? from each side** and type x when prompted.
2. Press **[ENTER]** to clean-up the result.
3. Move the 1 to the right-side. Press **[F4]** and choose **subtract...**
4. Press **[ENTER]** to clean-up the result.
5. Press **[F4]** and choose **divide...**
6. Press **[ENTER]** to clean-up the result.
7. Press **[PAGE DOWN]** repeatedly to scroll through your solution.

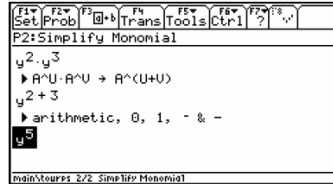


SMG Feature: As you have just seen, *SMG* will show the details of a transformation. To clean-up the result of a transformation without choosing a transformation from **[F4]**, press **[ENTER]**. This allows students to focus on the current new material without having to go through the details of previous work. If you want your students to handle the details themselves, have them choose appropriate transformations from **[F4]**.

8. To move to the next problem, press **[F6]:1**.

Problem 2: Simplify monomial $y^2 \cdot y^3$.

1. Press **[F6]:1** if you haven't already.
2. Press **F4** and choose $A^U \cdot A^V \rightarrow A^{(U+V)}$.
3. Press **ENTER** to clean up the previous result.
4. Press the up cursor to go up to the original math object $y^2 \cdot y^3$.
5. Choose a different transformation. The old transformation and result are replaced.



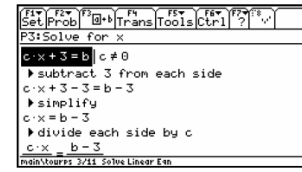
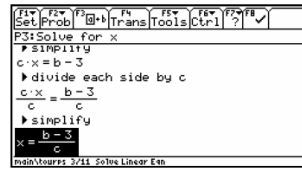
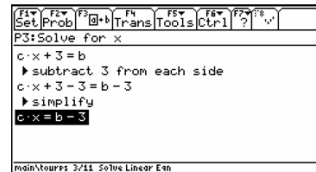
SMG Feature: SMG has 'quick keys' for certain frequently used transformations. For example press **+** to **add ? to each side** of an equation.

SMG Feature: SMG implements domain constraints. When a transformation changes the domain of definition, new conditions are displayed as part of the result.

6. To move to the next problem, press **[F6]:1**.

Problem 3: Solve linear equation $c \cdot x + 3 = b$ for x .

1. If you are not sure of the goal of solving this problem, press **[F7]:1**. We want to move the 3 on the left side to the right side.
2. Press quick key **+**. Type **3**. Press **ENTER**.
3. Press **ENTER**.
4. Press **+**. Type **c**. Press **ENTER**. Dialog box "This action ..." shows up.
5. Press **ENTER** to continue. Please note that the Problems statement has been modified to include the constraint $c \neq 0$.
6. Press **ENTER** to simplify the result.

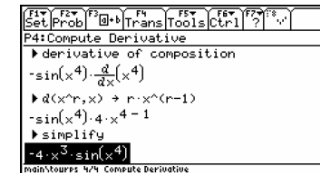
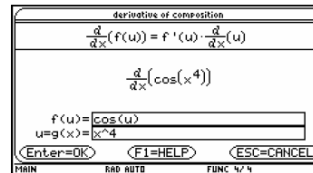
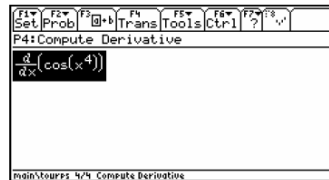


7. Press **[F6]:1** to move to the next problem in the current problem set.

Problem 4: Compute the derivative $\frac{d}{dx} \cos(x^4)$

1. Press **F4** and apply **derivative of composition**.
2. SMG provides a dialog box which requests definitions for the outside function, f , and the inside function, g . Enter the information or press **F1** for Help (SMG will provide possible choice for the two functions.) Make sure that the choice is the one you want. Change it if it is not. Press **ENTER** twice to continue to the display.
3. Think about the result (or write it down). Then press **ENTER** to see SMG's result.
4. Compare results and explain any significant differences.
5. Press **F4** and look for a way to transform the remaining derivative. Choose, for example,

$$\frac{d}{dx} x^r \rightarrow r \cdot x^{r-1}.$$



6. Press **F6**:1 to move to the next problem in the current problem set.

Next, consider several examples that use subexpression selection.

Experts solve algebra problems by recognizing that they can perform a certain transformation on a particular part of a math object and that this transformation moves toward the goal of the problem. For example, one might want to factor the left-hand side of an equation when trying to solve the equation.

REVISED MAIN PROCESS: For each step

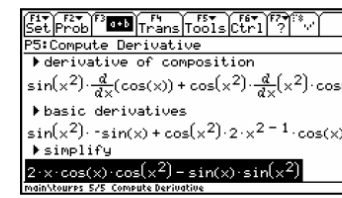
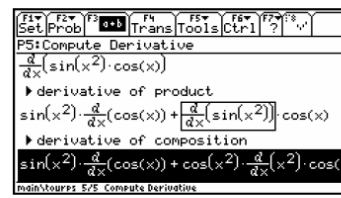
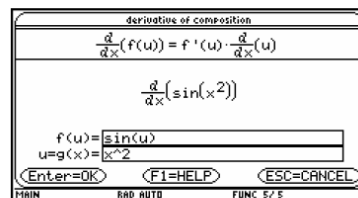
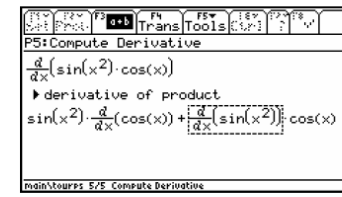
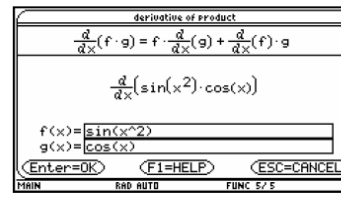
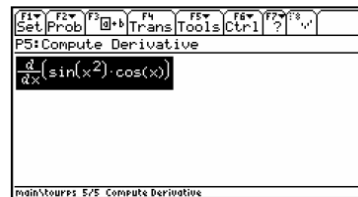
- A. Think about what part (or whole) of the object you want to change; select it.
- B. Press **F4** (Trans).
- C. If you see what you want to do, press the corresponding number.
- D. If you don't see what you want to do, choose a different part of the current math object. [See **SUBEXPRESSION SELECTION PROCESS** immediately below.]
- E. Press **F4** again to see what you can do with the currently selected math object.
- F. If you are not sure what you are supposed to be doing, check **F7**:1 for a hint or for the goal of this particular type of problem.

SUBEXPRESSION SELECTION PROCESS: [This information is available under [F7]:3.]

- To start press [F3] or [F4]: enter subexpr selection.
- To move to a smaller part of the object press \odot .
- To move to an object at the same level press \odot or \odot .
- To move to a larger object press \odot .
- To exit subexpression selection press [F3] or F4: exit subexpr selection.

Problem 5: Compute the derivative $\frac{d}{dx} \sin(x^2) \cdot \cos(x)$

- Press [F4] and choose **derivative of product**.
- SMG provides a dialog box which requests definitions for the functions f and g . Enter the information or press [F1] for Help (SMG will provide possible choices for the two functions.) Press [ENTER] twice to continue to the display.
- Press [F3]. Press \odot \odot to select $\frac{d}{dx} \sin(x^2)$.
- Press [F4] and choose **derivative of composition**.
- SMG provides a dialog box which requests definitions for the outside function, f , and the inside function, g . Enter the information or press [F1] for Help (SMG will provide possible choice for the two functions.) Make sure that the choice is the one you want. Change it if it is not. Press [ENTER] twice to continue to the display.
- Press [F4] and choose **basic derivatives**. Press [ENTER] to clean-up.



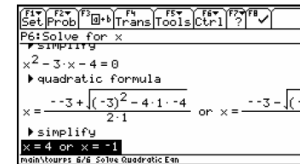
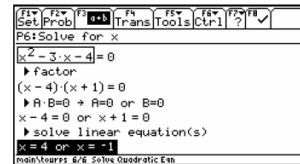
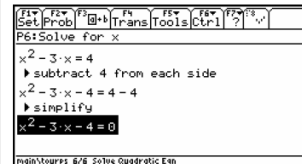
- Press [F6]:1 to move to the next problem in the current problem set.

Problem 6: Solve quadratic equation $x^2 - 3x = 4$.

- Press **[F3]** to try subexpression selection. Press \odot followed by \odot . Press \odot . Try your own arrow choices. Press **[F3]** or **[ESC]** to exit subexpression selection. If x^2 is selected, how do you select 4?
- Before trying to solve this problem, under **[F7]:1**, find the goal of solving quadratic equations.

Students should be able to factor the left-hand side of this equation once it is in the proper form.

1. Press **[=]** to **subtract...**, Type **4**. Press **[ENTER]** **[ENTER]**.
2. Press **[F3]** to select the left-hand side.
3. Press **[F4]** and choose **factor**. Note how the left side remains boxed so that you can tell what part of the equation was factored.
4. Press **[F4]** and choose **A·B = 0 -> A = 0 or B = 0**.
5. Press **[F4]** and choose **solve linear equation(s)**. Note that students don't need to figure out how to use *SMG* to finish this problem. A teacher could require students to provide the details, in which case they finish by selecting the individual equations and choosing appropriate transformations.



Let's try to solve this in a different way. The quadratic formula is useful for solving quadratic equations.

SMG Feature: Students can perform legal steps that are not necessarily the best steps and students can use different methods to solve a given problem. So *SMG* makes it easy for students to go back to any previous step and try a different transformation. Press \odot to get back to the step you wish to change. Then choose a new transformation.

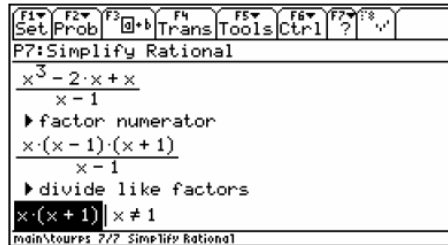
1. Press \odot until you get to the original equation. (You could use the 3rd equation.)
2. Check **[F4]** to see if we can use the quadratic formula now. It's not there. We need to modify the equation first.
3. Press **[=]** to **subtract...**, Type **4**. Press **[ENTER]** **[ENTER]**.
4. Now, choose **quadratic formula** from **[F4]**. Press **[ENTER]** to simplify the answers.

If you wish, you can go back to the first step, press **[2nd]** \odot and solve this quadratic equation by completing the square. (You are on your own.)

5. Press **[F6]:1** to move to the next problem in the current problem set.

Problem 7: Simplify rational expression $\frac{x^3 - 2x^2 + x}{x - 1}$.

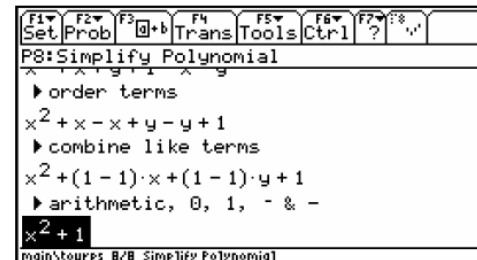
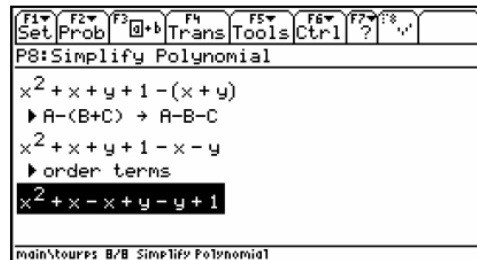
1. Press **F4** and choose **factor numerator**.
2. Press **F4** and choose **divide like factors**. Press **ENTER**.



3. Press **F6**:1 to move to the next problem in the current problem set.

Problem 8: Simplify the polynomial $x^2 + x + y + 1 - (x + y)$.

1. Press **F4** and choose **A-(B+C)->A-B-C**.
2. Press **F4** and choose **order terms**.
3. Press **F4** and choose **combine like terms**. Press **ENTER** again to clean-up.

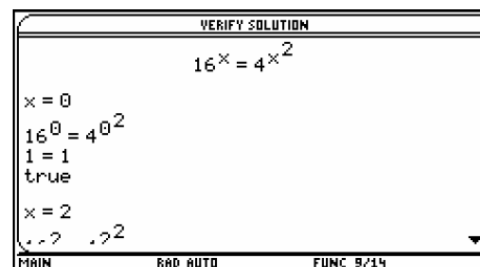
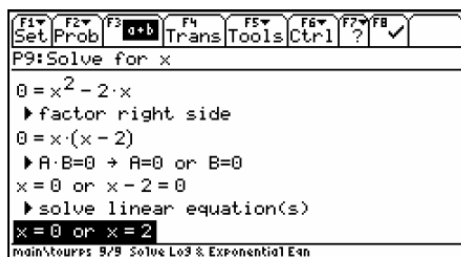
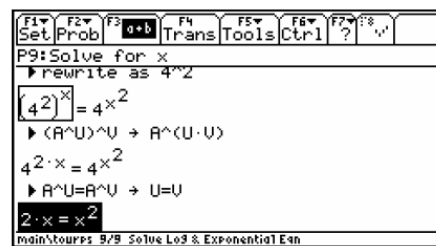
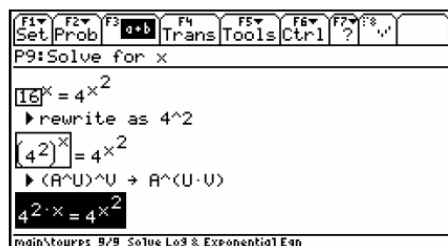


SMG Feature: To give the student flexibility to handle special situations, *SMG* allows students to select a subexpression and replace it with an equivalent expression. *SMG* will test for equivalence of the two expressions. Select the subexpression to change and press the **STO►** key.

4. Press **F6**:1 to move to the next problem in the current problem set.

Problem 9: Solve the exponential equation $16^x = 4^{x^2}$.

1. Press **F3**. Press \odot to select **16**. Press the **STO▶** key or choose **F7**: Rewrite. Type $4^{\wedge}2$. Press **ENTER**.
2. Press **F3** to select the left-hand side; press **F4** and choose $(A^{\wedge}U)^{\wedge}V \rightarrow A^{\wedge}(U^{\wedge}V)$.
3. Press **F4** and choose $A^{\wedge}U = A^{\wedge}V \rightarrow \dots$
4. Press **F4** and subtract $2x$ from each side. Press **ENTER**. Press **ENTER** again to clean-up.
5. Press **F4** and select **factor right side**.
6. Press **F4** and choose $A \cdot B = 0 \rightarrow \dots$
7. Press **F4** and choose **solve linear equation(s)**.
8. Press **F8** to verify solution.



SMG Feature: To give the student flexibility to handle special situations, *SMG* allows students to substitute variables for expressions. Select the subexpression to substitute for and press **F7**: Substitute

9. Press **ENTER** to return to the problem.
10. Press **F6**:1 to move to the next problem in the current problem set.

Problem 10: Simplify difference quotient $\frac{f(x+h) - f(x)}{h}$, where $f(x) = \sqrt{x}$.

1. Press **F4** and choose **apply definition of f**.
2. Press **F4** and choose **rationalize numerator**.
3. Press **F4** and choose **expand & simplify numerator**.
4. Press **F4** and choose **divide like factors**.

The calculator screen shows the following steps:

- Function: $f(x+h) - f(x)$
- Denominator: h
- Action: **▶ apply $f(x) = \sqrt{x}$**
- Result: $\frac{\sqrt{x+h} - \sqrt{x}}{h}$
- Action: **▶ rationalize numerator**
- Result: $\frac{(\sqrt{x+h} - \sqrt{x}) \cdot (\sqrt{x+h} + \sqrt{x})}{h \cdot (\sqrt{x+h} + \sqrt{x})}$

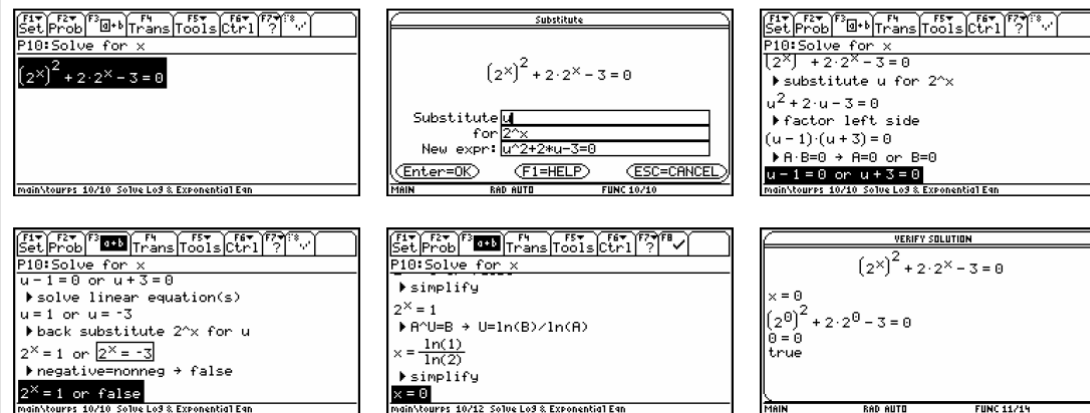
The calculator screen shows the following steps:

- Function: $h \cdot (\sqrt{x+h} + \sqrt{x})$
- Action: **▶ expand & simplify numerator**
- Result: $\frac{h}{h \cdot (\sqrt{x+h} + \sqrt{x})} \mid x+h \geq 0 \text{ and } x \geq 0$
- Action: **▶ divide like factors**
- Result: $\frac{1}{\sqrt{x+h} + \sqrt{x}} \mid h \neq 0$

5. Press **F6**:1 to move to the next problem in the current problem set.

Problem 11: Solve the exponential equation $(2^x)^2 + 2 \cdot 2^x - 3 = 0$.

1. Choose **F5**:Substitute.
2. SMG provides a dialog box which requests substitution information, definitions. Enter the information or press **F1** for Help (SMG will provide possible substitution information). Make sure that the choice is the one you want. Change it if it is not. Press **ENTER** twice to continue to the display.
3. Press **F4** and choose **factor left side**.
4. Press **F4** and choose **A·B=0 -> ...**
5. Press **F4** and select **solve linear equation**.
6. Press **F5** 4: Back Substitute to back substitute 2^x for u .
7. Press **F3**. Press \odot to select $2^x = -3$ and conclude that there are no solutions for this equation by pressing **F4** and selecting **negative = nonneg ->false**.
8. Pressing **ENTER** will get rid of “false”.
9. Press **F4** and choose **A^U = B -> ...** Press **ENTER** to clean-up.
10. Press **F8** to verify solution.

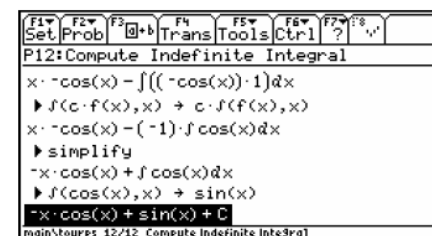
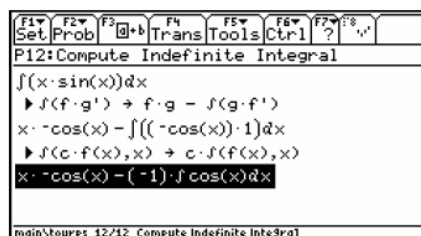
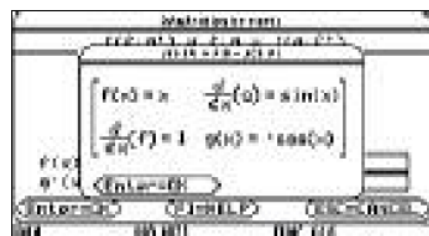
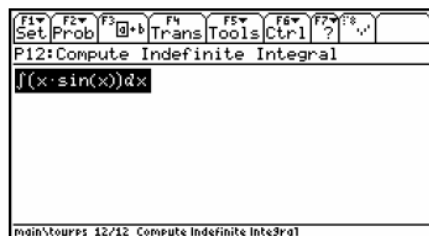


SMG Feature: Symbolic Math Guide lets you simplify expressions and solve equations that contain functions. When you create a problem that contains a function, a dialog box is displayed that lets you define the function. Transformation “Apply definition of ...” applies the definition of the defined function into the expression or equation.

11. Press **F6**:1 to move to the next problem in the current problem set.

Problem 12: Compute the indefinite integral $\int x \cdot \sin(x) dx$

1. Press **F4** and choose **integration by parts**.
2. SMG provides a dialog box which requests $f(x)$ and $g'(x)$. Enter the information or press **F1** for Help (SMG will provide possible choice for $f(x)$ and $g'(x)$). Make sure that the choice is the one you want. Change it if it is not.
3. Press **ENTER** until a dialog with $f(x)$, $f'(x)$, $g(x)$ and $g'(x)$ is displayed.
4. Press **ENTER**.
5. Press **F4** and choose $\int (c \cdot f(x)) dx \rightarrow c \cdot \int (f(x)) dx$.
6. Press **ENTER** to simplify.
7. Press **F4** and choose $\int \cos(x) dx \rightarrow \sin(x)$.



SMG Feature: The key SMG transformations to help students learn to solve indefinite integration problems are substitution, integration by parts and partial fractions. SMG also includes sum/difference and scalar product transformations as well as transformations for indefinite integrals of basic functions.