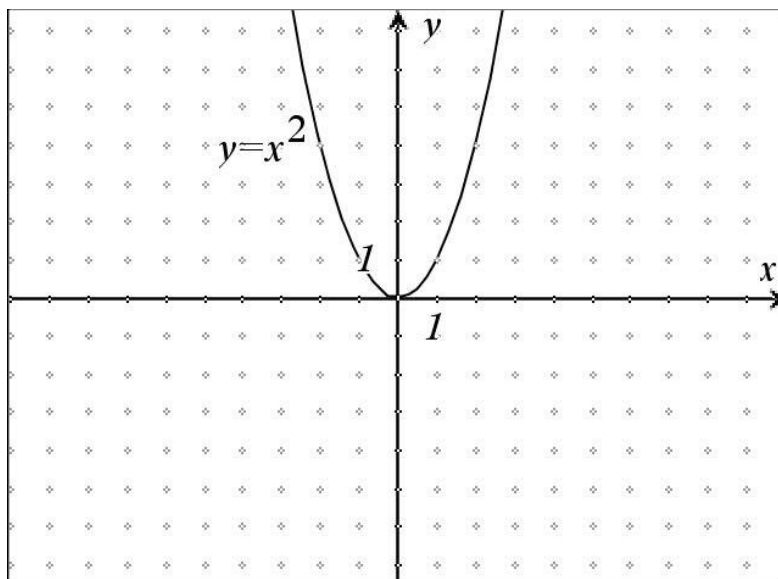


### Part 1 – Draw A Tangent Line by Hand

On the graph to the right, draw a line tangent to  $y = x^2$  in the first quadrant.

1. Approximate the slope of the line. Show your work.



2. Write the equation of your line.

### Part 2 – Draw and Explore Tangent Line with Technology

Press  $\left(\frac{\square}{\square}\right)$  and select **New Document**. (Unless you want to save what you are working on, arrow over to “No” and press  $\left(\text{enter}\right)$ .)

Select **Add Graphs** by using arrows and pressing  $\left(\text{enter}\right)$  or  $\left(\frac{\square}{\square}\right)$ .

Graph the equation  $y = x^2$  in **f1(x)** by pressing  $\left(\mathbf{x}\right) \left(x^2\right) \left(\text{enter}\right)$ .

Draw a tangent line on the graph in the first quadrant. Select **MENU > Points & Lines > Tangent**. Glide your finger across the Touchpad to use it as a mouse or click the arrows on the side of it to move the cursor to your desired point and press  $\left(\text{enter}\right)$ .

Zoom in and observe the behavior. Press **Menu > Window/Zoom > Zoom – In**. A magnifying glass with a plus sign will appear. Position this over the point of tangency and press  $\left(\text{enter}\right)$ . You have zoomed in by a factor of 2. Press enter a few more times over that point.

3. Write your observations of how your tangent line and the graph  $f1(x) = x^2$  compare when examined close up.
4. **Conjecture** – Will this type of behavior occur for all other functions? Explain your reasoning.

(You may want to try it for another function. You can choose your own function or try  $f2(x) = \sin(x)$  on another *Graphs* page. Press  $\left(\text{trig}\right) \left(\mathbf{x}\right) \left(\text{enter}\right)$  to graph  $\sin(x)$ .)

Press  $\left(\text{ctrl}\right) \left(\text{doc}\right) \left(\downarrow\right)$ , [*page*], to insert another page.

TIP: Press  $\left(\text{tab}\right)$  to go back to the entry line. Arrow up to see previously entered functions.)

# Local Linearity Discovery

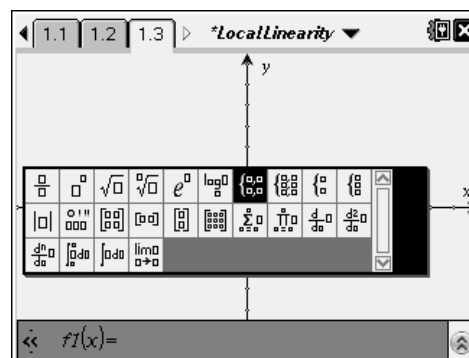
## Part 3 – Graph a piecewise function to explore local linearity

A function is said to be linear over an interval (i.e. locally linear over a small interval) if the slope is constant. Let's discover if all functions have a constant slope when they are examined in a small enough interval.

To graph  $y = \begin{cases} x^2, & x < 2 \\ 2x, & x \geq 2 \end{cases}$ , first insert another *Graph* page.

The piecewise function template can be found by pressing  $\left(\frac{\text{math}}{\text{math}}$ ). Arrow over to select the 2-piece piecewise function template and press  $\text{enter}$ . Then, press  $\left(\text{X}\right) \left(x^2\right)$   $\left(\text{tab}\right) \left(\text{X}\right) \left(\text{ctrl}\right) \left(=\right) \left(\text{enter}\right) \left(2\right) \left(\text{tab}\right) \left(2\right) \left(\text{X}\right) \left(\text{tab}\right) \left(\text{X}\right) \left(\text{ctrl}\right) \left(=\right) \left(\text{enter}\right) \left(2\right) \left(\text{enter}\right)$ .

Discover if all functions have the property of local linearity by zooming in on the point (2, 4). This point is called a *cusp*.



- Explain your observations. Use words like “slope” and “local linearity” to explain if, in the neighborhood of (2, 4), the function becomes one straight line. (Use **MENU > Window/Zoom > Window Settings** to ensure that your zoomed-in window still contains (2, 4). It may also help to show the end values of the axes if they are not already displayed; select **MENU > View > Show Axes End Values**.)

## Part 4 – Graph another piecewise function

To explore if all piecewise functions lack the property of local linearity, on a new *Graphs* page

zoom in on (2, 4) of the function  $f(x) = \begin{cases} x^2 & , x < 2 \\ 4x - 4, & x \geq 2 \end{cases}$ .

5. Does this function appear to be locally linear in the neighborhood of (2, 4)? Compare and contrast this function to the one graphed and explored in Problem 3.

## Part 5 – Conclusion

You know the definition of slope is  $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ . For the function  $f(x)$ , this can be written

as  $\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$ . If you were finding the slope of function in the interval of a

repeatedly zoomed in graph, describe what happens to  $\Delta x = x_2 - x_1$ .