AP® CALCULUS BC
2001 SCORING GUIDELINES

Question 5

Let f be the function satisfying $f'(x) = -3x f(x)$, for all real numbers x, with $f(1) = 4$ and $\lim_{x\to\infty} f(x) = 0$.

(a) Evaluate $\int_1^{\infty} 3x f(x)\,dx$. Show the work that leads to your answer.

(b) Use Euler's method, starting at $x = 1$ with a step size of 0.5, to approximate $f(2)$.

(c) Write an expression for $y \quad f(x)$ by solving the differential equation $\frac{dy}{dx} \quad 3xy$ with the initial condition $f(1) \quad 4$.

(a) $\int_1^{\infty} 3x f(x)\,dx$

$= \int_1^{\infty} f'(x)\,dx = \lim_{b\to\infty} \int_1^b f'(x)\,dx = \lim_{b\to\infty} f(x)\Big|_1^b$

$= \lim_{b\to\infty} f(b) \quad f(1) \quad 0 \quad 4 \quad 4$

2 : { 1 : use of FTC; 1 : answer from limiting process }

(b) $f(1.5) \approx f(1) - f'(1)(0.5)$

$= 4 - 3(1)(4)(0.5) = -2$

$f(2) \approx 2 + f'(1.5)(0.5)$

$\approx 2 \quad 3(1.5)(\ 2)(0.5) \quad 2.5$

2 : { 1 : Euler's method equations or equivalent table; 1 : Euler approximation to $f(2)$ (not eligible without first point) }

x	y	dy/dx
1	4	−12
1.5	−2	

$y_1 = 4 + (-12)(.5) = -2$

$y_{new} = y_0 + \frac{dy}{dx}\Delta x =$

(c) $\frac{1}{y}\,dy \quad 3x\,dx$

$\ln y \quad \frac{3}{2}x^2 + k$

$y = Ce^{\ldots}$

$4 = Ce^{\ldots}\ ;\ C = 4e^{\ldots}$

$y = 4e^{\ldots}e^{\ldots}$

5 : { 1 : separates variables; 1 : antiderivatives; 1 : constant of integration; 1 : uses initial condition $f(1) = 4$; 1 : solves for y }

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

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Question 6

A function f is defined by

$$f(x) = \frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \cdots + \frac{n+1}{3^{n+1}}x^n + \cdots$$

for all x in the interval of convergence of the given power series.

(a) Find the interval of convergence for this power series. Show the work that leads to your answer.

(b) Find $\displaystyle\lim_{x \to 0} \frac{f(x) - \frac{1}{3}}{x}$.

(c) Write the first three nonzero terms and the general term for an infinite series that represents $\int_0^1 f(x)\,dx$.

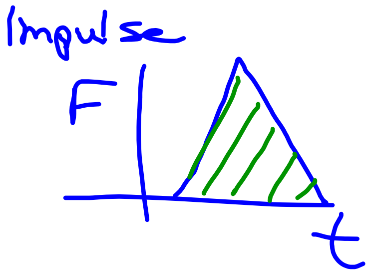
(d) Find the sum of the series determined in part (c).

Solution	Scoring
(a) $\displaystyle\lim_{n\to\infty}\left\lvert \frac{\frac{(n+2)x^{n+1}}{3^{n+2}}}{\frac{(n+1)x^n}{3^{n+1}}} \right\rvert = \lim_{n\to\infty}\left\lvert \frac{(n+2)}{(n+1)}\frac{x}{3} \right\rvert = \left\lvert \frac{x}{3} \right\rvert < 1$ At $x = -3$, the series is $\sum_{n=0}^{\infty}(-1)^n \frac{n+1}{3}$, which diverges. At $x = 3$, the series is $\sum_{n=0}^{\infty}\frac{n+1}{3}$, which diverges. Therefore, the interval of convergence is $-3 < x < 3$.	4 : 1 : sets up ratio test 1 : computes limit 1 : conclusion of ratio test 1 : endpoint conclusion
(b) $\displaystyle\lim_{x\to 0}\frac{f(x)-\frac{1}{3}}{x} = \lim_{x\to 0}\left(\frac{2}{3^2} + \frac{3}{3^3}x + \frac{4}{3^4}x^2 + \cdots\right) = \frac{2}{9}$	1 : answer
(c) $\displaystyle\int_0^1 f(x)\,dx = \int_0^1\left(\frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \cdots + \frac{n+1}{3^{n+1}}x^n + \cdots\right)dx$ $\displaystyle= \left(\frac{1}{3}x + \frac{1}{3^2}x^2 + \frac{1}{3^3}x^3 + \cdots + \frac{1}{3^{n+1}}x^{n+1} + \cdots\right)\Big\vert_{x=0}^{x=1}$ $\displaystyle= \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots + \frac{1}{3^{n+1}} + \cdots$	3 : 1 : antidifferentiation of series 1 : first three terms for definite integral series 1 : general term
(d) The series representing $\int_0^1 f(x)\,dx$ is a geometric series. Therefore, $\displaystyle\int_0^1 f(x)\,dx = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$.	1 : answer

$$\frac{\dfrac{n+2}{3^{n+2}}\,x^{n+1}}{\dfrac{n+1}{3^{n+1}}\,x^{n}}$$

$$= \frac{n+2}{n+1} \cdot \frac{3^{n} \cdot 3}{3^{n} \cdot 3^{2}} \cdot \frac{x^{n} \cdot x}{x^{n}} = \frac{n+2}{n+1} \cdot \frac{3}{9} \cdot x$$

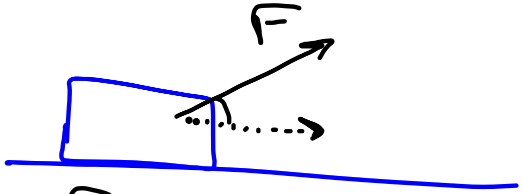
vp



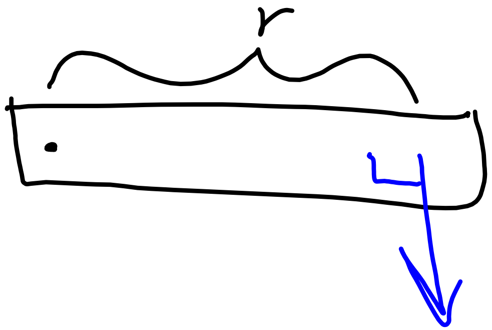
$$a = \frac{dv}{dt}$$

$$\int a\,dt = \int dv$$

$$\int a\,dt = \Delta v = v - v_0$$



$$\int F \cdot dr$$



$$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$$

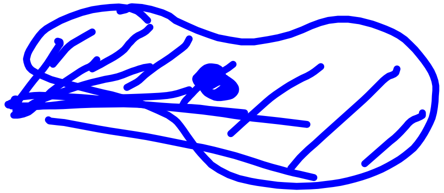
rotational Inertia

UNITS

$$I = \int r^2 dm = \sum mr^2$$

torque N.m

$$\mathbf{r}_{cm} = \sum m\mathbf{r} / \sum m$$

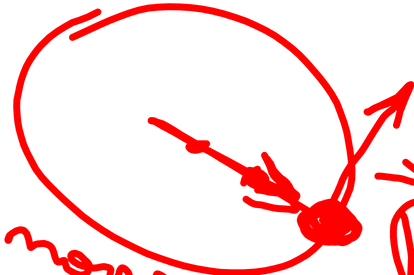


$$v = r\omega$$

kg.m² . rad/s²

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$$

ang momentum



$\vec{p} = m\vec{v}$

$\vec{L} = r m\vec{v}$

$$K = \frac{1}{2} I\omega^2$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\mathbf{F}_s = -k\mathbf{x}$$

$$U_s = \frac{1}{2}kx^2$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

May 2005

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

$$y = A\cos(bt - \phi) + C$$

$$y = a\sin(bx + ct) +$$

$$c = \frac{2\pi}{T} = 2\pi f$$

$$v = \lambda f$$

$$ac = \lambda \frac{2\pi}{c} \Rightarrow \lambda = \frac{ac^2}{2\pi}$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$\omega = \frac{2\pi}{T}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

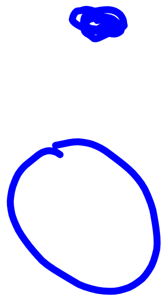
$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\mathbf{F}_G = -\frac{Gm_1m_2}{r^2}\,\hat{\mathbf{r}}$$

attr

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{qq}{r^2}$$

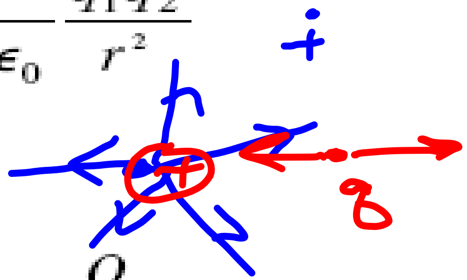
$$U_G = -\frac{Gm_1m_2}{r}$$

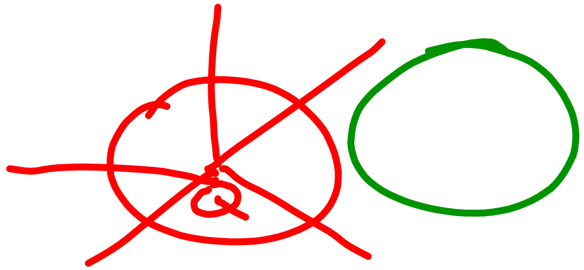


$$U_E = k\,\frac{qq}{r}$$

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$



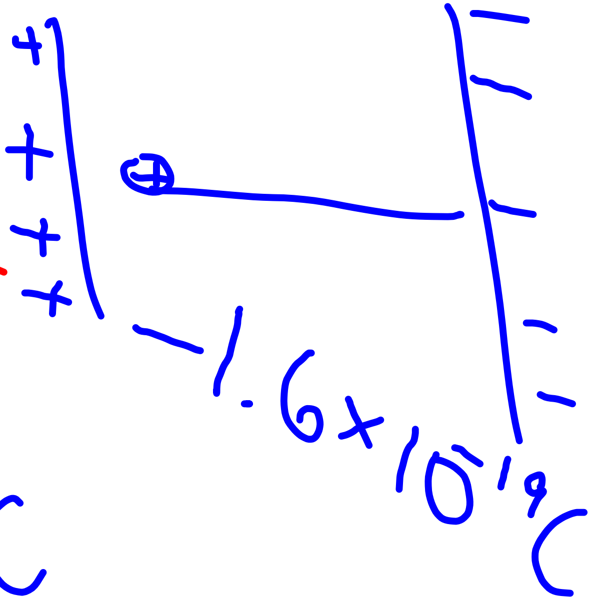


$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$E = -\frac{dV}{dr}$$

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

Scalar



$$U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

$U_E = qV$

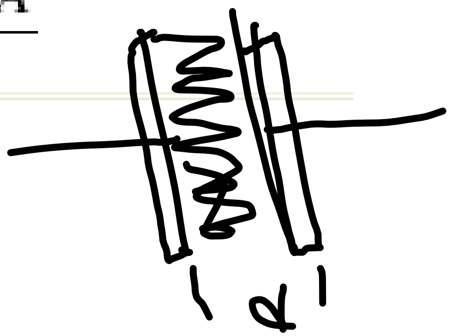
$-1.6 \times 10^{-19}\ C$

$\frac{J}{C}$

eV

$$C = \frac{Q}{V} \Rightarrow Q = VC$$

$$C = \frac{\kappa\epsilon_0 A}{d}$$



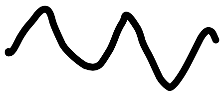
$$C_p = \sum_i C_i$$

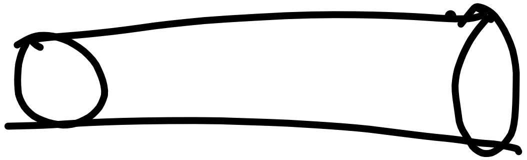
$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$I = \frac{dQ}{dt}$$

$$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$$

$$R = \frac{\rho \ell}{A}$$





$$V = IR$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$P = IV$$

$$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$$

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$$

$$\mathbf{F} = \int I \, d\boldsymbol{\ell} \times \mathbf{B} = BI\ell$$

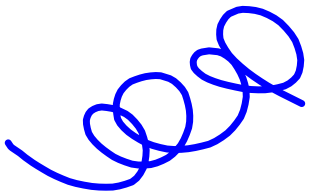
$$B_s = \mu_0 n I$$

$$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$$

$$\mathcal{E} = -\frac{d\phi_m}{dt}$$

$= IR$

$$\mathcal{E} = -L\frac{dI}{dt}$$



$$U_L = \frac{1}{2} L I^2$$