

STUFF YOU MUST KNOW COLD #2 CALC

Objective: Demonstrate that you know cold important basic information for the ap exam

Take the derivative of the following

1. $x^n \Rightarrow y' = n x^{n-1}$
2. $\sin 3x \Rightarrow y' = 3 \cos 3x$
3. $\cos 2 \Rightarrow y' = 0$
4. $\ln(\sec x) \Rightarrow y' = \frac{1}{\sec x} \sec x \tan x = \tan x$

5. $\csc x \Rightarrow y' = -\csc x \cot x$
6. $\csc^{-1}(x/3) \Rightarrow y' =$

$$\frac{-\frac{1}{3}}{\sqrt{\left(\frac{x}{3}\right)^2 - 1}} = \frac{-3}{|x|3\sqrt{\left(\frac{x}{3}\right)^2 - 1}} = \frac{-3}{|x|\sqrt{(x)^2 - 9}}$$

7. $\cot^{-1}(x-2) \Rightarrow y' = -1/(1+(x-2)^2)$
 $= -1/(x^2 - 4x + 5)$

8. $\cot^{-1} x^2 \Rightarrow y' = 2x \csc^2(-x^2)$

9. $3 \sec x \Rightarrow y' = 3 \sec x \tan x$

10. $\sec^{-1} x \Rightarrow y' = \frac{1}{|x|\sqrt{x^2 - 1}}$

11. $a^x \Rightarrow y' = a^x \ln a$

12. $\log_a x = \ln x / \ln a \Rightarrow y' = 1/(x \ln a)$

13. $\tan^{-1}(\tan x) = x \Rightarrow y' = 1$

14. $\cos^{-1} x \Rightarrow y' = -1/\sqrt{1-x^2}$

15. $e^{2x^5} \Rightarrow y' = 10x^4 e^{2x^5}$

16. $\sin^{-1} x \Rightarrow y' = 1/\sqrt{1-x^2}$

Evaluate the following

17. $\sin^{-1} 0 = 0$

18. $\sin 30^\circ = 1/2$

19. $\tan \pi = 0$

20. What is 30 degrees in radians? $\pi/6$

21. $\cos^{-1}(-1) = \pi$

Find the antiderivative

- 22.

$$\int \frac{1}{x^2 - 4x + 4} dx = \int \frac{1}{(x-2)^2} dx$$

$$\text{Let } u = x - 2 \Rightarrow \int \frac{1}{u^2} du = \int u^{-2} du$$

$$= -1(x-2)^{-1} + C$$

- 23.

$$\int_0^{\sqrt{2}} \frac{dx}{\sqrt{4-x^2}} = \int_0^{\sqrt{2}} \frac{\frac{1}{2}}{\frac{1}{2}\sqrt{4-x^2}} dx$$

$$= \frac{1}{2} \int_0^{\sqrt{2}} \frac{1}{\sqrt{1-(\frac{x}{2})^2}} dx \quad \text{Let } u = \frac{x}{2} \Rightarrow 2du = dx$$

$$= \int_{x=0}^{x=\sqrt{2}} \frac{1}{\sqrt{1-u^2}} du = \sin^{-1}\left(\frac{x}{2}\right) \Big|_0^{\sqrt{2}} = \frac{\pi}{4}$$

24. $\int \sec x dx = \ln|\sec x + \tan x| + C$

25. $\int -\sec x \tan x dx = -\sec x + C$

26. $4 \int \tan 2x dx = 2 \ln|\sec 2x| + C$

27. $\int_1^e \frac{dx}{2x} = \frac{1}{2} \ln x \Big|_1^e = \frac{1}{2}(1-0)$

- 28.

$$\int (x^3 - 1)^4 x^2 dx = \frac{1}{3} \cdot \frac{1}{5} (x^3 - 1)^5 + C$$

$$\text{Let } u = x^3 - 1$$

29. $\int_0^1 \frac{1}{2} x e^{x^2} dx = \frac{1}{4} (e^{1^2} - e^{0^2}) = \frac{1}{4}(e-1)$

$$\text{Let } u = x^2$$

30. State the Fundamental Theorem of Calculus

$$\int_a^b f(x) dx = F(b) - F(a)$$

where $F'(x) = f(x)$

31. State the Intermediate Value Theorem

If the fun $f(x)$ is continuous on $[a,b]$, and y is a number between $f(a)$ and $f(b)$, then there exists at least one number $x=c$ in the open interval (a,b) such that $f(c) = y$.

32. State the Chain Rule for Differentiation

$$\frac{d}{dx}[f(u)] = f'(u) \frac{du}{dx} \quad \text{OR} \quad \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

33. Symbolically write the Quotient Rule.

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{\frac{du}{dx} v - u \frac{dv}{dx}}{v^2} \quad \text{OR} \quad \frac{u'v - uv'}{v^2}$$

34. How do you find the critical points (get at least 2 of the 3 acceptable answers)?

$f' = 0$ or is undefined

be sure to check endpoints to see if they are extrema

35. What does an inflection point signify?

Change in concavity

36. Does the second derivative need to exist for there to be an inflection point?

NO, it could be undefined

37. How do you find a local maximum?

Deriv goes from neg to positive. Take der and set equal to zero (or undefined).

CHECK IT.

38. What are the critical points of

$$f(x) = (2x - 5)^3 (x + 4)^2$$

$$\begin{aligned} f'(x) &= 6(2x-5)^2 (x+4)^2 + 2(2x-5)^3 (x+4) \\ &= (2x-5)^2 (x+4) (6x+24+4x-10) \\ &= (2x-5)^2 (x+4)^2 (10x+14) \end{aligned}$$

Critical points occur at $x = 2.5, -4,$ and -1.4

$$39. \int \frac{-11}{(x+2)^2} dx = \frac{11}{x+2} + C$$

$$40. \int \sqrt{t^2 - 1} dt \quad \text{Bonus: Trig Substitution \& IBP}$$

$$\text{Let } \sec \theta = t \quad \tan \theta = \sqrt{t^2 - 1}$$

$$\sec \theta \tan \theta d\theta = dt$$

$$\int \tan \theta \sec \theta \tan \theta d\theta = \int (\sec \theta + \sec^3 \theta) d\theta$$

$$\ln |\sec \theta + \tan \theta| + \int \sec^3 \theta d\theta$$

$$\text{By IBP, } \int \sec^3 \theta d\theta = \frac{1}{2} \sec \theta \tan \theta - \frac{1}{2} \ln |\sec \theta + \tan \theta| + C$$

$$= \frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta| + C$$

$$= \frac{1}{2} t \sqrt{t^2 - 1} + \frac{1}{2} \ln |t + \sqrt{t^2 - 1}| + C$$

41. Take the derivative of $\sqrt{t^2 - 1}$

$$\rightarrow y' = \frac{1}{2} (t^2 - 1)^{-\frac{1}{2}} (2t) = \frac{t}{\sqrt{t^2 - 1}}$$

42.

$$\int_0^1 7^x dx = \frac{7^x}{\ln 7} \Big|_0^1 = \frac{7-1}{\ln 7}$$

$$= \frac{6}{\ln 7}$$

43. State the MVT (Mean Value Theorem).

If the function $f(x)$ is continuous on $[a,b]$, and it is also differentiable on the open set (a,b) you can always find an $x = c$ in (a,b)

$$\text{such that } f'(c) = \frac{f(b) - f(a)}{b - a}$$

44. How do you find the average velocity?

$\frac{\text{final position} - \text{initial position}}{\text{total time}}$ or

$$= \frac{\int_{t_i}^{t_f} v(t) dt}{t_f - t_i}$$